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External Examiner's Report on Doctor of Philosophy Thesis of Mrs. Anna Karmazyn entitled „Synchronization of coupled mechanical oscillators in the presence of noise and parameter mismatch "

1. Introduction

Reviewed work consists of six chapters. In the first, introductory one, major ideas involved in the field of synchronization of chaotic systems are reviewed, and several types of synchronization features are presented in detail. In the second chapter, classification and properties of the coupling between dynamical systems are presented. Next chapter presents the theoretical background necessary for the analysis of dynamics of the nonlinear oscillators. Stability of synchronization states, the algorithm of the Lyapunov exponents and transversal Lyapunov exponents is discussed and idea of the master stability function allowing synchronization stability test is presented. In Chapter 4 a method of studied systems modelling is shown. The content of this Chapter is a numerical analysis of the oscillators network with physical pendula attached to these oscillators. Two particular but representative cases of one ($n = 1$) and three ($n = 3$) oscillators have been considered. The system with introduced parameter's mismatch is also numerically analysed. The fifth chapter concerns a fundamental part of work i.e. discussion and analysis of conducted experiments results. Last, chapter eight, contains a summary of general conclusions and perspectives of future research in thesis subject matter.

2. Ph.D thesis evaluation

A literature review and theoretical background presented in the chapters 1 to 3 concern different possibilities of analysis of synchronous motions and their transitions to a state of desynchronization. This discussion is complete, critical and serves Ph. thesis Author to original research tasks and scientific thesis formulation. The aim of the dissertation is an identification of main transition from synchronous motion to a state of desynchronization, as a result of external disturbances and small differences between parameters of nominally identical coupled mechanical oscillators. I think that numerical analysis and experimental research and analysis of kinematically excited coupled mechanical oscillators could be a subject of a Ph.D thesis.
Valuable is the chaotic synchronization of pendulums revealed in case of $n=1$. Crucial question for an explanation of chaotic synchronization mechanism in the system is to identify the nature of coupling between the pendula. A complexity of this coupling is visualized by matrix form. Possible synchronous responses, also in chaotic regime, are an effect of direct inertial (first component with inertial coupling matrix), nonlinear diffusive coupling or they can be also caused by external forcing. As a result, there appear complex form of the interaction between the pendula, which can lead to chaos synchronization. Characteristic for this coupling configuration is a mutual interaction between vibrating bodies and pendula, so independent subsystem cannot be extracted (isolated) from this structure of the oscillator.

Therefore, this scheme of coupling cannot be treated either as a master-slave unidirectional connection or as an autonomous driver or active-passive decomposition.

Numerical analysis of the considered system demonstrates that due to vertical direction of forcing, angular oscillations and rotations of pendula are activated only in the direct neighbourhood of principal resonance frequency of the system. Outside this range of excitation frequency we observe an extinction of angular oscillations of pendula and system is reduced to 1DoF linear oscillator executing vertical vibration. However, close to resonance high amplitude of the mass excites nonlinear response of pendula and transition nonlinear dynamics takes place. One can observed complete synchronization in phase or in anti-phase, which are equivalent coexisting solutions. Chaotic synchronization state is characterized with one positive Lyapunov exponent. Loss of stability of chaotic synchronization is caused by chaos-hyperchaos transition, when second Lyapunov exponent becomes positive. Thus, this exponent play a rule of transversal or conditional Lyapunov exponent quantifying the stability of synchronous regime. Other reason of observed desynchronization can be possible coexistence of attractors, even in the W-range of regular motion, e.g., one pendulum oscillates while the second is stabilized in upper or lower position. On the other hand, for increased mass of main oscillator chaotic synchronization of pendula does not occur. Possible explanation is that increasing mass influences for decrease of coupling terms in the system under consideration. Hence, coupling rate is reduced when the mass grows. Then pendula "lose mutual contact", which manifests with independent chaotic or hyperchaotic motion of both pendula.

Equally remarkable is numerical analysis of the system with introduced parameter's mismatch, which results in interesting type of synchronous behaviour manifesting with alternately appearing and disappearing states of chaotic synchronization in-phase and anti-phase. This effect was called intermittent phase – anti-phase synchronization. Such dynamical behaviour can be explained by a presence of a specific memory of the principal modes of vibration. In phase and in anti-phase oscillations are typical principal modes of linearized system.

In Chapter 5 numerical results for $n=1$ have been verified experimentally qualitatively and in general quantitatively. Phenomena of in phase, in anti-phase and intermittent phase – anti-phase transition have been observed, registered and analysed during experimental study. Comparison of numerical and experimental outcomes for $n=3$ also demonstrates a qualitative their agreement. In this augmented system chaotic synchronization of pendula has not been detected in numerical simulations and in experiment. However, various synchronous configuration have been observed. These results indicate existence of cluster synchronization between pairs of pendula. Experimental studies have confirmed the results obtained numerically, which proves the correctness of the adopted model. During the analysis, particular attention was paid to study the phenomenon of synchronization between pendula suspended on the elastic beam. This phenomenon has been detected both in numerical simulations and during the experiment. Moreover, it has been observed in the simplest case of one 3DoF oscillator (mass with two pendula, $n=1$) as well as for three such oscillators (9DoF system, $n=3$) located on the beam.

Summarizing the evaluation of Ph.D thesis, I consider that the dissertation contains original elements of a scientific task and could be a basis for obtaining a technical science doctor's degree. Thesis presented on the 11th page of discussed work is proved.

Only one critic consideration is connected to the description of the main theoretical model (Page. 7-9) and the model of experimental rig (Page 77-78). There are some editorial errors.

3.Final Recommendations

My evaluation of the PhD dissertation entitled "*Synchronization of coupled mechanical oscillators in the presence of noise and parameter mismatch*" by *Mrs. Anna Karmazyn* is positive. I highly value the content of the thesis, its editorial structure and its usefulness for the higher education in Poland. The thesis constitutes both original and substantial contributions to the advancement of knowledge in the subject chosen. The author proved herself to be a competent researcher, and I highly value her knowledge in the area of applied mechanics, in mathematical modelling of mechanical systems, in experimental and simulation methods and in the theory of non-linear vibrations. Therefore, I recommend that the degree of Doctor of Philosophy in the field of Engineering be awarded to Mrs. Anna Karmazyn on the basis of the above refereed dissertation. Opinioned PhD dissertation corresponds with conditions connected to the Act on Academic Degrees and Scientific titles from 12th of September 1990 with changes from 14th of March 2003, hence allows public defense of previously mentioned thesis.